numbers + magic = answer

Students explaining: Make the most of mental computation

ANNALIESE CANEY

provides questioning techniques to help teachers 'capture' the mathematical ideas of children as they explain their mental computation strategies.

> 'I just see the numbers, and I just sort of fiddle around with them in my head and then get the answers there.' (Grade 6 student)

> 'Well I half know and half don't and sometimes I just guess!' (Grade 5 student)

Facilitating mental computation discussion comes with its own set of challenges for researchers and teachers. It is important to be able to help capture the students' ideas, before they disappear and seem like little more than moments of magic!

In enabling children to learn how numbers work, the current emphasis on mental computation is driven by the principles of constructive learning. It is here that questions of 'how' and 'why' become particularly important for both teachers and students. When students are working mentally, simple questions posed to an individual or a class of students, such as, 'How did you work this out?' and, 'Why does that work?' allow us to peer into the world of number and gain an understanding from the perspective of the students. It also encourages students to re-evaluate their own ideas when exposed to the thinking of their peers. Experiences that encourage discussion and learning are far removed from the emphasis on activities that focus on testing that have dominated mental computation in primary and secondary classrooms for so long. Many teachers rightly feel that testing activities of some sort still have a place in the classroom, for example, as a quick way of gauging what types of questions students are able to cope with. However, as the sole approach to teaching mental computation, they really are a thing of the past.

Talking in depth to students is often thought to be the privilege of a researcher, but increasingly teachers are being encouraged

to develop critical thinking by probing how their students think and learn. These 'mental moments', which I have collected from interviews with students, are intended to interest those teachers wanting to make the most of mental computation in their classrooms. How do you access what went on in the seconds between the end of your question and a student answering? What types of questions are helpful to ask?

Moving past — Is it right?

When students work mentally through problems they often give their responses and promptly ask, 'Is that right?'. This one question can shift the focus of the student (and the class) — the strategies they used and their ideas simply disappear. Students who persist with this type of question often are right but are cautious in proceeding without some sort of confirmation. A classroom environment that focusses only on obtaining answers is not conducive to getting students comfortable in talking about numbers and sharing their ideas. An environment that fosters inquiry by means of discussion is, and certainly needs some consideration, when setting up your mental computation time. Students must know that all types of responses are valued, including partial attempts — where students might be able to solve just some of the problem, and approximate answers. Students also need to be encouraged to take responsibility for their responses; sometimes this involves getting them to go back through the problem and see if they still get the same answer. Ask students to explain how they checked as this can be different from how they originally worked.

Moving through an incorrect answer

When students do give incorrect answers I always try to get the student to talk back through what they did. Often they will identify their own mistakes in the process particularly if this involves 'slip ups' rather than conceptual misunderstandings. Consider for example, the following two dialogues.

Grade 5

Interviewer: 24 + 8.
Student: 34.
Interviewer: 34 — OK. How did you do this one?
Student: I just thought of it.
Interviewer: Just thought of it. Tell me more about how you thought of it, the 24 + 8?
Student: Well I just sort of added them up to equal to 34.
Interviewer: Right — added them up very fast or did you use some number facts?
Student: Just used some number facts — 10.
Interviewer: How did that fit in?
Student: Well I just added 24 + 10 and you take away 2 from that.
Interviewer: OK so 24 + 10 and how much was that?
Student: 34 — took away 2 and that should equal 32! [grinning]

Grade 9

Interviewer: Half of 78?
Student: Um… 36.
Interviewer: How did you do that?
Student: I don't know, I just thought I knew it.
Interviewer: Right, well I said half of 78, so how would you check your answer?
Student: Um… I'd probably just see: I'd get my answer that I put and then I'd add it again onto it.
Interviewer: Right, so what was your answer?
Student: 36.
Interviewer: Right, so what would you do?
Student: I'd do 6 plus 6, equals 12, so and I'd put the one up and then that'd equal 7 — so I got it wrong!
Interviewer: Alright, but what do you have to do to get it right? You had 72 and I gave you 78, so what do you think?
Student: I get it up some more and I choose 39. Because 9 plus 9 equals 8, so it must have been that because it can't have been 49.

It is important to emphasise that these students had good strategies; they just did not quite make all the necessary adjustments!

I don't know! I can't remember! I just guessed!

As shown in the quotes from students at the beginning of this paper, students often can not seem to remember how they worked through a problem or do not seem to be able to communicate what went on between hearing the question and reaching an answer. Some students need a little more prompting than others and I found that it was very useful to ask students, 'Which number were you thinking about first?' or, 'Tell me about the very first thing you did'. Persuading the student to tell you just one thing will often facilitate in remembering lost strategies and answers. Consider these two examples.

Grade 4

Interviewer:	24 × 3.
Student:	72.
Interviewer:	Well done, how did you do that one?
Student:	Um… I've forgotten
Interviewer:	What was the very first thing you thought of doing?
Student:	I added up all the twenties, and it equals 60.
Interviewer:	So you added up all your twenties first and got 60.
Student:	And then I added up the 3 fours… then it equals 72.
Interviewer:	Good. Now with your 3 fours did you know what 3 fours were or did you add them together?
Student:	I know 2 fours was 8, so 8 plus 4 equals 12.

Grade 4

Interviewer:	125 – 89.
Student:	36. I'm not sure!
Interviewer:	Why did you say 36?
Student:	I just guessed it I think!
Interviewer:	What would be the first thing you'd think of doing?
Student:	Well there's 1 left over… 11 left over in 100 because there's 89. There's another 10 and you've got 1 more before that 10, so that's 11. You add the 11 on the 25 which equals 36.

As I suspected, there was a little more to these responses than good guesses!

Handling an unusual response

Consider the subtraction problem 125 – 99. As you might expect, in solving this problem mentally, there was a variety of explanations from the students, such as '99 is 1 before 100 so I added it onto 25' (Grade 3).

Then along came Erin (Grade 6): 'I took the 25 from 99 which is 74, then I took the 74 from 100 which is… 26'. This was a strategy I had never encountered before but it worked! Further questioning, 'Why did you take 25 from 99?' revealed a little more about her thinking: 'Because it's easier to work with hundreds because we have got to take the 25 off the 125 so we can work with the hundred. So we're left with 74. Then we take the 74 out of the 100. That leaves us with 26.' Erin attempted several related problems to see if she used the same strategy, which she did. Interestingly she also endeavoured to complete a written algorithm for this problem and seemed to find this task difficult and confusing. Erin was very pleased to hear that her mental strategy was unique.

I just knew it!

Often this is true of course; although in interviewing students I am reluctant to let any opportunity slip by! First, it is always worth asking again and second, I would also encourage students to think about how they would explain that problem to another student who was finding it difficult.

This can be a good opportunity to ask students to check their answers, particularly if they sound a little unsure. For example, for the question 12 × 10, Emily (Grade 5) said it was 'hard to explain' because she just knew the answer. In asking if she had a way of checking her answer she replied: 'I

know that 12 tens are 120 because you just like add up 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, that's 12, except I didn't do it like that, I just knew it.' Again it is useful to emphasise that her explanation might be helping somebody else in the class.

Working aloud

It is not always necessary to get students to work problems out in their heads first, and then explain; encourage students to work aloud through problems from start to finish. Kelly, in Grade 5, muttered under her breath a lot when working on 24 × 3. I kept hearing 48 but her response did not seem to include 48: '72: I added the three 20s, which is 60 and added 3 fours which is 12.' I asked her about the 48 and she replied, 'Yeah, I tried to do it that way: 24 + 24, 48 and it was a bit hard that way so then I tried another way and added the three 20s which is 60 and then I got the three 4s.' Kelly really worked hard on this problem, exploring several alternative strategies. This is an excellent example that was nearly missed! After talking with Kelly I often give students the option of working aloud if they prefer or sometimes I just ask students to work this way, particularly if they appear confused.

Getting students to work aloud has its own advantages: being able to follow students through problems enables teachers to gain further insight into where students hesitated, ran into problems, or changed strategies, and this can be very interesting. It will also send the message to the other children in the class that there are many different ways to work things out and that a strategy that works for one person might not work so easily for another. Moreover, as Yazujian (2002) points out, this 'may provide a way for our students to change from passive observers to active participants' in the classroom (p. 108).

More than numbers — Opportunities for exploring language

It can be challenging to run with student thinking at the best of times but in discussing numbers with students, do not forget to listen to the language they use. In Tasmania the outcome of 'being numerate' is situated within 'communicating,' one of the five *Essential Learnings* that frame the curriculum (Department of Education Tasmania, 2002). For students, developing the confidence to voice their ideas will improve their communication skills as well as enhance their competence in working with numbers. Here are some examples of responses from students that made our discussions very interesting!

Interviewer:	What about 22 × 10?
Student:	Is 220. Dissect the 10 and then you times 22 by the 1, then you add the zero.
Interviewer:	100 – 34.
Student:	You take 50 away and then another 26 away?
Interviewer:	So 100 take 50 and then you got 50…
Student:	Then you take 34 away from the other 50… and then you squeeze what's left with the 50, so add it on to the other 50.
Interviewer:	170 – 70.
Student:	100
Interviewer:	Right, why?
Student:	Because you're just like taking 70, so you just have to wipe it off.
Interviewer:	125 – 99.
Student:	26. I just cut the 100, get 25 and then add 1, 26 because its 99.
Interviewer:	The first question is 16 + 8.
Student:	What do you mean, just like plopping them together?

Notice from the above examples, that the students use language that describes actions. Ask students to explain the phrases that they use or hear and make comments on what 'I just cut the 100' might actually mean.

Encouraging flexibility — Asking related questions

Emily, a Grade 5 student, struggled at first with 7 × 3, answering 24 and eventually making it to 21 by knowing that 2 × 7 was 14 and adding on another 7. Yet 7 × 6 seemed much easier for her as she answered, '42: that was plus the 21 to 21'. Emily went on to explain that it was a much quicker way 'instead of having to count all the way up to it'.

Again with the problem 7 × 3, Sam (Grade 4) used the same strategy as Emily to solve 7 × 3, but for 7 × 6, he decided to add three 14s explaining 'add up the units and then you add up the tens but I do that back to front'.

These were occasions where the questions were related and it can be very interesting to see how the students adapt their strategies, or are able to use something they have just worked out to solve another problem. For example, Laura (Grade 4), first solved half of 62 by doing 'half of 60 is 30 and half of 2 is 1' but then used what she had just worked out to solve half of 78, 'It equals 39. Half of 60 is 30 and half of 10 is 5 so you put a 5 and then you add a 4'. Asking related questions can provide an opportunity to see if students have one strategy that they will stick to or if they demonstrate a certain degree of flexibility depending on the numbers and the task they are presented with.

More often than not it seems that mental computation is not about finding and following a strategy but finding a starting point: 'I just had a mental block. I just had a lot of numbers going round in my head and I'm just trying and trying to work it out.' This type of comment was not uncommon. Sometimes students appear to get caught up in trying to remember a way to solve a problem rather than concentrating on the numbers and operations at hand. Threlfall (2001) stresses that 'flexibility will arise consequentially through the emphasis on considering possibilities for numbers rather than by focussing on holistic strategies'.

Conclusion

It is worth remembering some of the characteristics that have been used to describe mental algorithms (Plunkett, 1979). These include *variable*, *flexible*, and also *constructive* in that working mentally involves building a solution path that may depend inherently on the properties of numbers involved — of which there are many. He also describes them as *fleeting* in that they can be 'difficult to catch hold of' — so maybe just a little bit magical!

'I just have weird ways of working some things out.' (Grade 7 student)

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